$$E(x,t)=\frac{\gamma}{\mu}p_0(x)\exp\left\{\frac{\gamma}{\mu\beta}\left[\int_0^t\int_\Omega (u^2+pv)\,dx\,d\tau+B(x,t)\right]\right\}$$

$$B(x,t)=[\mu]\int_0^t U\,d\tau-\int_\Omega v(y,t)\int_y^x u(z,t)\,dz\,dy-\int_\Omega v_0(x)\int_0^x u_0(y)\,dy\,dx+\beta\int_0^x u_0(y)\,dy$$

Once we have time-uniform estimates for the density, all the remaining estimates, as well as the asymptotic forms with respect to time can be obtained by following the same line of reasoning as in /3/.

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A METHOD OF CALCULATING THE AERODYNAMIC CHARACTERISTICS OF BODIES ON THE BASIS OF INVARIANT RELATIONS OF THE THEORY OF LOCAL INTERACTION*

A.V. ANTONETS and A.V. DUBINSKII

The existence of relations between the aerodynamic characteristic of solids of revolution of various forms that are invariant to the model of the flow over them and to the angle of attack is proved. A method of calculating the characteristics is developed on that basis. An example of its use for bodies with a generatrix of exponential form is considered, and a comparison with "exact" numerical calculations is made.

Within the framework of models of local interaction (/1-4/ and others) the local interaction force of the flow at each point of the body surface depends only on the local angle of attack and on parameters that define the process of flow over bodies "as a whole". Such models are effectively used over a wide range of flows (the free molecular mode, hypersonic flows of dense and rarefied gas, the light stream flow, and the intermediate region of rarefied gas flow). However, existing methods of aerodynamic calculations (/1,4/ and others) presume a knowledge of the specific model of local interaction.

Let the surface of a convex solid of revolution in the system of coordinates $x\varphi r$ attached to the body be given by the function $r(x)$ with the Ox axis directed along the body axis. An expression for the coefficient of the projection of the aerodynamic force $\mathbf{R}$ on some direction defined by the unit vector $\mathbf{l}$ can be represented in the form

$$C_l=\frac{\mathbf{R}\cdot\mathbf{l}}{qS_k}=\frac{1}{S_k}\int_0^{r_k}\int_0^{2\pi}F_l\left(\alpha,\varphi,\frac{dr}{dx}\right)d\varphi\cdot r\,dr=\int_{u_-}^{u_+}\Phi_l(\alpha,u^{-1})\frac{d}{du}\left(\frac{r}{r_k}\right)^2du,\quad q=\frac{\rho_\infty v_\infty^2}{2}\tag{1}$$

where q is the pressure head, α is the angle of attack, and S_k and r_k are the area and radius of the middle cross section. The functions F_l, Φ_l depend on the indicated arguments and the model of the flow, and $u=dx/dr$ is the cotangent angle of inclination of the body contour to its axis that takes values from u_- to u_+.

Let us consider $n+1$ bodies whose generatrix angle of inclination to the axis varies over the same range. The subscript ν indicates the number of the body. Then, if the function $r_\nu(u)$ $(\nu=0,1,\ldots,n)$ satisfies the condition

$$\left(\frac{r_0}{r_{k0}}\right)^2-\sum_{\nu=1}^n\beta_\nu\left(\frac{r_\nu}{r_{k\nu}}\right)^2=C\tag{2}$$

it follows from (1) that their AX $C_{l\nu}$ of the same kind are connected by the relation

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$$C_{l0} = \sum_{\nu=1}^{n} \beta_\nu C_{l\nu} \tag{3}$$

This relation is invariant to the choice of the function Φ_l, which is the same for all bodies, i.e. the relation holds for any model of local interaction and, unlike the case of two three-dimensional bodies in /5/, for any angle of attack and any component of the aerodynamic force. Since we consider bodies whose generatrices start from the origin of coordinates, we set $C = 0$.

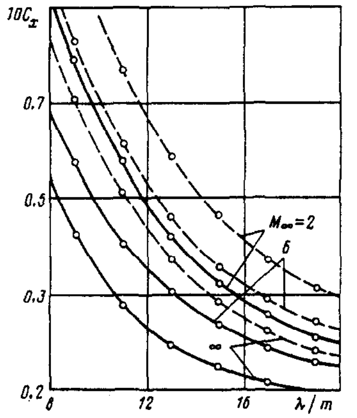

Fig.1

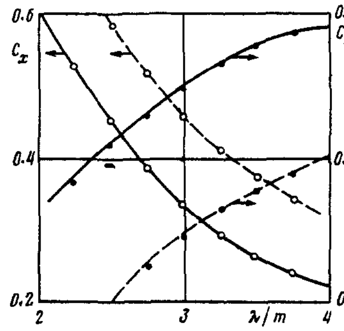

Fig.2

Hence on the basis of information about the drag and lift coefficients $C_{x\nu}(\alpha, u_+), C_{y\nu}(\alpha, u_+)$ of several "basic" bodies, by varying the parameters β_ν we can obtain the respective characteristics $C_{x0}(\alpha, u_+)$, $C_{y0}(\alpha, u_+)$ of a wide class of configurations. The calculation is based only on the assumption of the local nature of the interaction of the flow with the body surface, and does not require a knowledge of the specific model of local interaction, i.e. it enables us to calculate the aerodynamic characteristics under conditions, when the information necessary to determine the forces acting on the surface of the bodies is not available, and other methods of aerodynamic calculation are inapplicable.

Let the $C_{l\nu}(\alpha, u_+)$ of basic bodies be known, as before, and suppose it is required to determine the $C_{l0}(\alpha, u_+)$ of a body of given form. Naturally a β_ν that will ensure that condition (2) is satisfied does not generally exist. However, if one interprets β_ν as a coefficient of expansion of the function $r_0^2(u)/r_{k0}^2$ with respect to the basic functions $r_\nu^2(u)/r_{k\nu}^2$, it is possible in the majority of cases to expect an acceptable accuracy when calculating the characteristics using Eqs.(3), with the advantage that the procedure is relatively simple.

The practical importance of this approach lies in the fact that it can be used when other methods of determining the characteristics, that do not require a knowledge of the specific flow model, are not available. Various methods based on interpolation cannot generally be considered as alternatives owing to the non-availability of a parameter with which it can be realized. Note that the idea of the method in /5/ for solving "straight" problems of finding the characteristics of bodies of a given form was suggested earlier by A. I. Bunimovich and G. G. Chernyi.

As an example, consider the case when all the bodies are of "exponential" form, i.e.

$$\frac{r_\nu}{r_{k\nu}} = \left(\frac{x_\nu}{L_\nu}\right)^{m_\nu} = \left(\frac{u}{u_k}\right)^{t_\nu}, \quad t_\nu = \frac{m_\nu}{1 - m_\nu}, \quad \lambda_\nu = \frac{L_\nu}{r_{k\nu}}$$

The choice of a class of bodies is determined by the availability of systematic published results of "axact" numerical calculations /6/, which together with the additionally determined non-symmetric flows over bodies, provide a vast amount of information suitable for analyzing the accuracy of the proposed method, and confirming the invariance of relations (3).

Using the method of least squares to find β_ν in the segment $0 \leqslant u/u_k \leqslant 1$ we obtain a set of linear algebraic equations whose solution has the form

$$\beta_\nu = \frac{1 - m_0}{1 - m_\nu} \left(\prod_{\substack{i=1 \\ i \neq \nu}}^{n} \frac{m_0 - m_i}{m_\nu - m_i} \right) \prod_{i=1}^{n} \frac{1 + m_i + m_\nu - 3m_i m_\nu}{1 + m_i + m_0 - 3m_i m_0}, \quad \nu = 1, 2, \ldots, n \tag{4}$$

Thus in the case of bodies of exponential form simple explicit formulas exist for calculating the coefficient β_ν, that are independent of the flow model and the angle of attack, for which the characteristic is considered, and also of the parameter u_+, which is the same for all bodies. Hence for a particular class of bodies of exponential form the calculation of the characteristics using Eqs.(3) and (4) is preferable to the method of interpolation with respect to m.

Taking as basic bodies with exponents $m_1 = 0.5$, $m_2 = 0.6$, $m_3 = 0.7$, for $m_0 = 0.55$ and $m_0 = 0.65$ and using formula (4) we obtain, $\beta_1 = 0.374$, $\beta_2 = 0.758$, $\beta_3 = -0.134$ and $\beta_1 = -0.121$, $\beta_2 = 0.734$, $\beta_3 = 0.389$ respectively.

Curves of the functions $C_x(\lambda/m)$ are shown in Fig.1, calculated using the results obtained in /6/ for a Mach number $M_\infty = 2.6, \infty; \gamma = 1.4$ of the oncoming flow, and those calculated using the method described in /7/ are presented in Fig.2 for bodies of exponential form with a spherical nose (this deformation does not require a recalculation of β_ν) for three-dimensional flow of an ideal gas over a body $\alpha = 10°$, $M_\infty = 20$, $\gamma = 1.4$. The small circles and dots correspond to recalculations using Eq.(3), the solid lines correspond to $m_0 = 0.65$, and the dashed lines to $m_0 = 0.55$.

It can be seen that the determination of aerodynamic calculations using relations (3) enables us to obtain estimates of the aerodynamics force components that are very close to the exact calculations for the supersonic flow of an ideal gas over a body. This result applies also to the flow of an equilibrium and non-equilibrium dissociating gas. For actual gas flows the non-dependence of the coefficients β_ν on the flow conditions over the body, the angle of attack and, also, of which aerodynamic force component is considered, is confirmed.

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EQUILIBRIUM IN A CUT ALONG AN ARC OF A CIRCLE IN THE CASE OF INHOMOGENEOUS INTERACTION OF THE EDGES*

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Equilibrium in a cut along the arc of a circle is considered for the case of biaxial tension-compression. Under such a stress a free surface forms along the cut, and a zone of adhesion and mutual displacement appears in the region of contact when frictional forces are present. A non-singular solution is constructed for this case at the boundary of the zone of contact and free surface, and of the zone of adhesion and mutual displacement. Earlier, the problems of the free surface as well as the region of contact were considered in /1-3/. A solution was found in /4/ for a cut along the arc of a circle in a complex state of stress for the case when the edges interact at the extension of the cut, taking into account the formation of the adhesion and displacement zones.

1. Consider a cut along the arc of a unit circle. The equation of the cut in x, y-coordinates is $x = \cos\theta$, $y = \sin\theta$, $\alpha_0 \leqslant \theta \leqslant \beta_0$ (α_0, β_0 are the coordinates of the cut boundary). We have at infinity the mutually perpendicular stresses p, q ($p \leqslant 0$, $q \geqslant 0$) and p makes an angle γ with the Ox-axis. We shall describe the stress state using the complex Kolosov-Muskhelishvili potentials $\Phi(z)$, $\Psi(z)$ /1/

$$\sigma_r + \sigma_\theta = 2[\Phi(z) + \Phi^*(z)] \tag{1.1}$$

*Prikl.Matem.Mekhan., 47, 5, 874-880, 1983.